Inverse problems for wave equations with the Dirichlet and Neumann cavities

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In this talk, a simple mathematical model for an inverse problem by using waves is discussed. The target cavities D are in a homogeneous medium. Set a region B outside of $\overline{D}$. We put incident waves on $\overline{B}$ and catch waves reflected by the boundary ∂D of D on the same place $\overline{B}$. We can get measurement data by this procedure. Using this one pair of data, we can find the shortest time that waves traveling from a point in ∂B to a point in ∂D by using the enclosure method introduced by M. Ikehata of Hiroshima University.

In inverse problems for time dependent problems, the approach using the enclosure method is developed by many people. For the heat equations, the enclosure method clarified the way of finding the distance between objects and place taken data. This fact is shown by the author and Ikehata. Our problems discussed here are formulated by the wave equations. For the case of the wave equations, Ikehata intensively investigates by using some elliptic estimates and a fundamental solution of the reduced problem formulated as an elliptic equation with large parameter. This approach has an advantage of regularity for ∂D. Instead, we need to assume that all boundaries of separated cavities are governed by the same boundary condition: all are the Dirichlet boundary condition, or the Neumann boundary condition. This situation may be called monotonicity condition since this setting gives some monotonicity for the indicator function which is introduced in the formulation of the enclosure method. Thus, our next purpose is how to handle the case that this monotonicity is not satisfied.

In this talk, we introduce a simple attempt by using asymptotic solutions. If all of the boundaries are C^2 class and the Neumann boundary condition, we can give the distance between B and D. If we also have the boundaries with the Dirichlet boundary condition, we need to put more regularities of the Dirichlet boundary. Since we use the asymptotic solutions, we can treat the combined problem with the Dirichlet and the Neumann coditions like a linear combination of each problem.

This is a joint work with Wakako Kawashita of Hiroshima University.

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